THERMAL-HYDRAULIC STABILITY ANALYSIS OF A NATURAL CIRCULATION BASED BWR

Naveen Kumar*, A. K. Nayak and P. K. Vijayan

*Reactor Engineering Division, Bhabha Atomic Research Centre, Trombay, Mumbai–400085, India

INTRODUCTION

The Advanced Heavy Water Reactor (AHWR) is a light water cooled and heavy water moderated pressure tube type boiling water reactor. The reactor is designed with the twin objective of utilization of abundant thorium resources and to meet the future challenges to nuclear power such as enhanced safety and reliability, better economy and proliferation resistance. In AHWR, it is proposed to remove the core heat by natural circulation during start-up, power raising, normal operation, transients and accidental conditions. AHWR uses several passive concepts with a view to simplify the design and to enhance safety. Since the development of boiling water reactors, the thermal hydraulic stability has been one of the most important issues concerning the nuclear reactor designers. No instability incident took place for several years of BWR operation. However, since then, a number of fuel modifications have been imposed and core power densities have also been increased. One of the consequences of these modifications has been the increased vulnerability of BWRs to instabilities. Several such incidents have been reported in the literature since early 80s. Dauria et al (2004) has compiled a number of such incidents, starting from Coarso (Italy) in 1982 to Oskarshem (Sweden) in 1999. Instabilities are undesirable for several reasons. While on one hand, instability can result in oscillations of the power and flow rate, which may pose considerable amount of difficulties in reactor control, on the other hand, sustained flow oscillations may induce mechanical vibration of components. Flow oscillations may also cause premature CHF (critical heat flux). Two-phase natural circulation systems are susceptible to different types of flow instabilities like Ledinegg instability, flashing, Gysering, density wave oscillations. Among these, density wave oscillations are probably the most common and most widely studied type of instabilities encountered in two-phase flow systems. Density wave instability results from the multiple feedbacks between the flow rate, the vapour generation rate and the pressure drop in a boiling channel. The most important modes of density wave instability are loop and parallel channel instabilities. The parallel channel mode corresponds to a system of a big number of channels connected in parallel, in which a constant pressure drop condition governs flow through each of the channels. Experimentally, both in-phase and out-of-phase oscillations are observed in parallel channels. However, in-phase oscillation is a system characteristic and parallel channels do not generally play a role in it. With in-phase oscillation, the amplitudes in different channels can be different due to the unequal heat inputs or flow rates. Occurrence of out-of-phase oscillations is characteristic of parallel channel instability. The phase shift of out-of-phase oscillations is known to depend on the number of parallel channels. With two channels, a phase shift of 180° is observed. With three channels, it can be 120° and with five channels it can be 72°. With n-channels, Aritomi et al. (1986) reports that the phase shift can be 2π/n. However, depending on the number of channels participating, the phase shift can vary anywhere between π and 2π/n. For example, in a 3-channel system one can get phase shift of 180° or 120° depending on whether only two or all the three channels are participating. Aritomi et al. (1986) noted that in a parallel channel system, the flow instability first occurs in the most unstable pair of channels. Similar observations were made by Nayak et al. (1998) for a system having identical parallel channels. However, in a pressure tube type boiling water reactor, all the channels having different layouts and hence different geometries. Also because of inherent variation in neutron flux from core centre to periphery, these channels encounter different power. The present paper studies the behaviour of such systems.
MATHEMATICAL MODEL

Two general approaches are possible for stability analysis: linear stability analysis and non-linear stability analysis. The threshold of stability can be predicted using linear stability analysis method. Nayak et al. (1998) used linear stability analysis for predicting the behaviour of natural circulation system. The linear analysis method, sometimes also referred to as frequency domain analysis is used to determine the threshold of instability. The method involves linearization of governing equations around steady state operating points. The linearised equations are then perturbed (Laplace transformed) around their steady state to obtain the characteristics equation. The stability of the system is then predicted from the roots of the characteristics equation.

For this purpose let us consider the main heat transport system of a BWR as shown in Fig. 1 shows the schematic of main heat transport system of AHWR. Shown in the figure are a number of coolant channels connected between a common inlet header and a common steam drum. The steam drum and inlet header are joined by downcomers. The subcooled liquid enters the reactor core at the bottom and as it rises through the core, it gets converted into two-phase mixture. The steam-water mixture then rises through the risers which join at the bottom of steam drum. In the steam drum, separation of steam and water takes place. The steam goes to the turbine and an equal amount of subcooled water (known as feed water) enters into the steam drum. This subcooled liquid then flows through the downcomers to the inlet header completing the circulation path.

![Schematic of Main Heat Transport System of a AHWR](image)

Figure 1: Schematic of Main Heat Transport System of a AHWR

The conservation equations of mass, momentum and energy for one-dimensional two-phase flow are given by

\[ A \frac{\partial \rho}{\partial \bar{t}} + \frac{\partial \bar{w}}{\partial \bar{x}} = 0, \]

(1)
The equation of state is
\[ \rho = f(p, h). \]

The governing equations are linearised by superimposing small perturbations of \( w', h', p' \) and \( v' \) over the steady state values as follows
\[ w = w_{ss} + w' \]
\[ h = h_{ss} + h' \]
\[ p = p_{ss} + p' \]
\[ v = v_{ss} + v' \]

The perturbed conservation equations can then be written as
\[ -A \frac{\partial \dot{v}}{\partial t} + v_{ss}^2 \frac{\partial \dot{w}}{\partial x} = 0 \]
\[ \frac{1}{A} \frac{\partial \dot{w}}{\partial t} + \frac{w_{ss}}{A^2} \left[ w_{ss} \frac{\partial}{\partial x} (v') + 2 \frac{\partial}{\partial x}(w' v_{ss}) \right] + \frac{gw'}{v_{ss}} + \frac{fw_{ss}}{2DA^2} (w_{ss} v' + 2v_{ss} w') + \frac{\partial \dot{p}}{\partial x} = 0 \]
\[ A \frac{\partial \dot{h}}{\partial t} + w_{ss} v_{ss} \frac{\partial \dot{h}}{\partial x} = \begin{cases} -\frac{w' v_{ss}}{w_{ss}} q_h A & \text{heated region}, \\ 0 & \text{adiabatic region}. \end{cases} \]

The equations (4), (5) and (6) are then Laplace transformed which yield the following results
\[ -A s \dot{v}(s) + v_{ss}^2 \frac{\partial \dot{w}(s)}{\partial x} = 0 \]
\[ \frac{sw'(s)}{A} + \frac{w_{ss}}{A^2} \left[ w_{ss} \frac{\partial}{\partial x} (v'(s)) + 2 \frac{\partial}{\partial x}(w'(s) v_{ss}) \right] + \frac{gw'(s)}{v_{ss}} + \frac{fw_{ss}}{2DA^2} (w_{ss} v'(s) + 2v_{ss} w'(s)) + \frac{\partial \dot{p}(s)}{\partial x} = 0 \]
\[ Ah'(s) + w_{ss} v_{ss} \frac{\partial \dot{h}(s)}{\partial x} = \begin{cases} -\frac{w'(s) v_{ss}}{w_{ss}} q_h A & \text{heated region}, \\ 0 & \text{adiabatic region}. \end{cases} \]

The equations (7), (8) and (9) are solved for single-phase and two-phase regions of the loop and integrated to obtain the perturbed pressure drops. For each channel connected between header and steam drum, the perturbed pressure drop can be written as
\[ (\Delta p')_{H-SD} = G_1(s) (w'_m(s))_1 = G_2(s) (w'_m(s))_2 \]
It may be noted here that $h'(s)$ and $v'(s)$ can be obtained as function of $w'(s)$.

The perturbed pressure drop for the single phase region between the steam drum and the inlet header is given by

$$(\Delta p')_{SD-H} = G_T(s)(w_T'(s))$$ \hspace{1cm} (11)$$

Since, for the case considered, the total pressure drop across the system is zero, we can write

$$(\Delta p')_{SD-H} + (\Delta p')_{HSD} = 0$$ \hspace{1cm} (12)$$

For the core wide instability, the total perturbed pressure drop across the system should remain constant i.e. equation (12) must hold. This implies

$$G_T(s)(w_T'(s)) + G_1(s)(w_m'(s))_1 = 0$$ \hspace{1cm} (13)$$

Further, the total perturbed flow rate in the single phase region must be equal to the sum of the perturbed flow rates at the inlet of all the parallel channels i.e.

$$w_T'(s) = (w_m'(s))_1 + (w_m'(s))_2$$ \hspace{1cm} (14)$$

Using the techniques of linear system control theory, the core wide instability can be considered as a feedback system. The block diagram for the same is shown in Fig. 2.

![Block diagram for in-phase instability model](image-url)

Using control theory, it can be noted from the above block diagram that

$$w_T'(s) = \left( \frac{1}{G_1} + \frac{1}{G_2} \right) \left( 1 + \frac{G_T}{G_1} + \frac{G_T}{G_2} \right) (\Delta p')_T$$ \hspace{1cm} (15)$$
The equation, \( 1 + \frac{G_T}{G_1} + \frac{G_T}{G_2} = 0 \) is called the characteristic equation. The stability of the system is determined by the nature of the roots of this characteristic equation. The mathematical formulation for both, in-phase and out-of-phase instability remains same, however, two differ only in the nature of boundary conditions. In out-of-phase instability (also known as parallel channel instability), the downcomers does not participate in the oscillations.

\[ i.e. \ w'_T(s) = 0 \] (16)

The block diagram for feedbacks acting in a parallel channel system can be represented as shown in Fig. 3.

Using control theory, it can be noted from the above block diagram that

\[ (\Delta p'_{H-SD})_T = \frac{G_i}{1 + \frac{G_T}{G_2}} w'_T(s) \] (17)

Equation, \( 1 + \frac{G_1}{G_2} = 0 \) is called the characteristic equation. The stability of the system is determined by the nature of the roots of this characteristic equation.

**METHOD OF ANALYSIS**

As discussed earlier, for a multi-channel system the number of oscillation modes can be very large depending upon the number of channels participating. For a system having 113 channels, there can be 6328 combinations for 2nd mode of oscillations. With three channels participating, the number of combinations need to analysed increases to 234136. The problem is fairly simplified for a system having identical channels and identical layout. Nayak et al (1998) showed that for such a system, the stable zone is governed by the second harmonics. However, for an actual reactor system, all channels differ in their layout and hence are not identical. Fig. 4 shows the typical power map for one quarter of the AHWR core. All the channels having similar layout (identical lengths) are shown in one colour. It is seen from Fig 4 that from layout considerations, all the channels can be grouped into 12 groups. However, the problem needs to be simplified further. In the present study, 4 representative channels were selected for analysis. These are M13, N1, H8 and E5. The selection criterion will become clear as the discussion progresses.
RESULTS AND DISCUSSION

Fig. 5 shows the stability map for 2nd mode of oscillation for channel M13, N1, H8 and E5. Parallel channel instability is strongly influenced by the ratio of single-phase pressure drop component to two-phase pressure drop component. While single-phase pressure drop has a stabilizing effect, two-phase pressure drop has destabilizing effect on the system behaviour. Table 1 gives the ratio of single-phase to two-phase lengths for the channels considered. AHWR core has 1/8th geometrical symmetry. As we move from core centre towards periphery single-phase to two-phase length ratio increases. Hence the stable domain also increases. This is clear from Fig. 5 also. It is seen from Fig. 5 that channel N1 and channel E5 have the largest stable operating domain, followed by channel H8 and channel M13. This is because of the above reason that these channels have been selected for analysis.

Table 1: Ratio of single-phase to two-phase lengths for representative channels

<table>
<thead>
<tr>
<th>Channel</th>
<th>Single-phase length</th>
<th>Single-phase length/two-phase length</th>
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<tr>
<td>M13 (Refereed as Ch. 1)</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>N1 (Refereed as Ch. 2)</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>H8 (Refereed as Ch. 3)</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>E5 (Refereed as Ch. 4)</td>
<td>0.5</td>
<td></td>
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</table>

In a multi-channel system, channel powers are different and any two channels may oscillate in 2nd mode of oscillation. Therefore, the system needs to be analysed for all possible modes of oscillation. In the present study, the ch. 1, 2, 3 & 4 (please refer Table 1 for nomenclature) were analysed for the following 15 possible modes of oscillations. In all these predictions, the channel power was assumed to be varying in the ratio in which they are in equilibrium core (refer Fig. 4). Table 2 gives the results of analysis. It is seen from Table 2 that 3rd and higher modes of oscillation die out much before the 2nd mode of oscillation. Even for the 2nd mode of oscillation also, the stability threshold has been found to be lower than that for the same mode of oscillations for identical twin channels.
Table 2: Results of stability analysis (U-Unstable, S-Stable)

<table>
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<tr>
<th>Normalized Power</th>
<th>1234</th>
<th>134</th>
<th>124</th>
<th>234</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>23</th>
<th>24</th>
<th>34</th>
<th>11</th>
<th>22</th>
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<td>S</td>
<td>S</td>
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CONCLUSIONS

A methodology has been presented for analysing the stability behaviour of a multi-channel natural circulation system having different channel layouts. The proposed methodology has been applied to Advanced Heavy Water Reactor (AHWR) and the stable zone of operation for the reactor has been presented.

Nomenclature

\( A \) \hspace{0.5cm} \text{area of cross section, m}^2

\( D \) \hspace{0.5cm} \text{pipe diameter, m}

\( f \) \hspace{0.5cm} \text{Fanning friction factor, dimensionless}

\( G \) \hspace{0.5cm} \text{Transfer function}

\( g \) \hspace{0.5cm} \text{gravitational acceleration, m/s}^2

\( H \) \hspace{0.5cm} \text{Transfer function}

\( h \) \hspace{0.5cm} \text{specific enthalpy (kJ/kg)}

\( P \) \hspace{0.5cm} \text{perimeter (m)}

\( p \) \hspace{0.5cm} \text{pressure, Pa}

\( q \) \hspace{0.5cm} \text{Heat flux, W/m}^2

\( t \) \hspace{0.5cm} \text{time, s}

\( v \) \hspace{0.5cm} \text{Specific volume (m}^3/\text{kg)}

\( x \) \hspace{0.5cm} \text{running coordinate, m}

\( w \) \hspace{0.5cm} \text{mass flow rate, kg/s}

Subscripts

\( h \) \hspace{0.5cm} \text{heater}

\( H-SD \) \hspace{0.5cm} \text{Header – steam drum}

\( SD-H \) \hspace{0.5cm} \text{Steam drum - Header}

\( ss \) \hspace{0.5cm} \text{Steady state}

\( T \) \hspace{0.5cm} \text{Total}

REFERENCES